PHYC/OCEA 4412/5412

Assignment #4

Due: Thursday, 8 April 2010, in class

1) a) Determine the polarization relations for Rossby waves. Express the perturbation speeds, relative vorticity and stretching vorticity in terms of perturbation geopotential. Assume the reference density is constant for simplicity.

b) Draw diagrams in the x-y and x-z planes illustrating the structure of a Rossby wave with meridional wavenumber l=0.

c) Draw another diagram in the x-y plane illustrating the structure of a Rossby wave, but with wavenumbers k=l.

2) Surface waves in deep water are governed by the wave equation

$$\nabla^2 p' = 0$$

which given in geometric coordinates and is also known as Laplace's equation. Using the same setup as for shallow water waves, the boundary conditions are

$$\frac{\partial p'}{\partial z}\Big|_{z=-H} = 0$$

and

$$p'(z=\eta) = \rho g \eta$$

a) Using separation of variables, show how to obtain the solution

$$p' = p_0 \left\{ \frac{a}{2} \exp[\kappa_H (z+c)] + \frac{b}{2} \exp[-\kappa_H (z+d)] \right\} \exp[i(kx + ly - \omega t)]$$

from Laplace's equation.

b) Rewrite the solution using

$$a = b = \frac{\rho g \eta_0}{p_0 \cosh(\kappa_H H)}$$

and

$$c = d = H$$

Show that this choice of parameters allows the boundary conditions to be satisfied.

BONUS QUESTIONS:

c) Compare the solution you obtained with that for shallow water waves.

d) The dispersion relation for deep water waves is

$$\omega^2 = g\kappa_H \tanh(\kappa_H H)$$

Show that in the long wavelength limit the shallow-water result is obtained for both the solution and the dispersion relation.

e) Show in the long wavelength limit that long wavelengths progress faster than short wavelengths.

f) Show in the long wavelength limit that the magnitude of the group velocity is half that phase speed.