# PHYC/OCEA 4412/5412

## Assignment #3

Due: Thursday, 11 March 2010, in class

1) The "loudness" of a sound in decibels is given by

$$dB = 20 \log_{10}(p_0/p_r) \quad ,$$

where $p_0$ is the pressure wave amplitude and $p_r = 20\,\mu Pa$ (the threshold of human hearing at 1 kHz frequency) is the reference pressure. Determine the number of decibels required for one of the non-linear terms in the momentum equation to be important. Is this sound level in the normal range of human hearing?

2) This question will lead you through a perturbation analysis of surface gravity waves in shallow water, such as are seen in a bath tub. Take the tub bottom to be at depth $z = -H$ and the basic-state surface altitude to be at $z = 0$. Let the surface altitude perturbation be $z = \eta(x, y, t)$. For boundary conditions, take the surface pressure to be zero (i.e., ignoring the atmospheric pressure so that $p(z = \eta) = 0$ ) and the vertical speed at the bottom be zero ( $w(z = -H) = 0$ ).

IMPORTANT: Clearly break down your answer into parts a), b), c) etc, as I do below.

a) Draw a diagram of the problem, and mark z, x, H and $\eta$ on it.

b) Write down an appropriate set of governing equations. Explain your assumptions.

c) Define each variable as a sum of basic states and perturbations. Explicitly state the dependence for each variable.

d) Derive the basic state equations, and show $p'(z = \eta) = \rho g \eta$.

e) Show that the perturbation equations of motion are

$$\frac{\partial u'}{\partial t} = -\rho^{-1} \frac{\partial p'}{\partial x}$$

$$\frac{\partial v'}{\partial t} = -\rho^{-1} \frac{\partial p'}{\partial y}$$

$$\frac{\partial p'}{\partial z} = 0$$

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial x} + \frac{\partial w'}{\partial z} = 0$$

and

$$w'(z = \eta) = \frac{\partial \eta}{\partial t} \quad .$$

Hint: For the last equation, you should start by writing down an equation that says the vertical speed at the surface is just the total derivative following the motion of a parcel on that surface.

f) What does the perturbation hydrostatic equation say about the spatial structure of $p'$?

g) We need to start combining the equations together to eliminate perturbation variables. Show that

$$\frac{\partial}{\partial z}\frac{\partial w'}{\partial t} = \rho^{-1}\nabla_H^2 p'$$

h) Integrate both sizes of the equation in part (g) with respect to altitude between the bottom and the surface. Then show that

$$\frac{\partial^2 \eta}{\partial t^2} = gH\nabla_H^2 \eta$$

i) Solve for $\eta$ and determine the dispersion relation, assuming that the wave can progress in any (horizontal) direction.

BONUS QUESTIONS:

j) Find and compare the shallow water phase speed and group velocity for the wave with l=0. Provide an interpretation for the behaviour you find.

k) Determine the polarization relations in terms of the surface perturbation $\eta$. Note: You may choose any of the equations you found along the way for this purpose.

l) Show the “anatomy” of a water wave in a diagram.