PHYC 2050

Assignment #3

Due: Monday, 15 March 2010

1) The logistic law of population growth is

$$\frac{dp}{dt} = ap - bp^2$$

where p is the population, t is time, and a and b are coefficients. The first term on the right-hand side is responsible for population growth when resources are plentiful, and the second term limits the population as consumption makes resources scarce.

a) Derive a finite difference solution for the logistic law.

b) Write a program that uses your finite difference formula to model the historical population growth of England. Estimates since 1066 can be found at http://aolab.phys.dal.ca/~tomduck/temp/pop/. Read in the data using the appropriate function in scipy. Plot the modeled and actual values from 1066 through 2250. The parameters that produce the best fit after the industrial revolution are $p(1066) = 7.99$, $a = 1.92 \times 10^{-2}$ yr^{-1}, and $b = 3.42 \times 10^{-10}$ yr^{-1}.

c) How well does the model agree with the observed values? Propose explanations for any deviations between the two.

2) The probability density P(v) for molecular speeds v in a gas is given by Maxwell's distribution

$$P(v) = Av^2 \exp(-Bv^2)$$

where A and B are parameters depending on the mass, temperature, and gas constant. Suppose A = 3.663 x10^{-8} s^3/m^3 and B = 6.41x10^{-6} s^2/m^2.

a) Plot P(v) between 0 and 1200 m/s.

b) Use the Brent Algorithm to find the speed with the maximum probability density, v_{max}.

c) Calculate total probability for molecular speeds less than v_{max}. Hint: You will need to numerically integrate the probability density.

d) Calculate and plot the accumulated probability curve

$$A(u) = \int_0^u P(v)dv$$

for values of u between 0 and 1200 m/s. Hint: This will require you to make many calculations like you did in part (c).