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The spectrum of waves and turbulence at the tropopause

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[1] Aircraft measurements of winds, temperatures, and composition were conducted with a high-altitude aircraft in the tropopause region above Northern Wales during May and June 2000. Observations on 6 June 2000 along three vertically stacked horizontal flight legs show fluctuations due to both gravity waves and turbulence. Horizontal kinetic energy spectra determined from the measurements extend to 3 m in wavelength, the smallest scales yet available at these heights. The vertical wind spectrum for strong turbulence is considerably different in both shape and intensity from the non-turbulent spectrum. A sharp knee in the turbulent spectrum at 100 m wavelength is associated with the equipartition of kinetic energy in three dimensions. At other scales the vertical motions are relatively suppressed. A wave with 500 m wavelength was found, and provides a transition between the turbulent and non-turbulent cases. **Citation:** Duck, T. J., and J. A. Whiteway (2005), The spectrum of waves and turbulence at the tropopause, *Geophys. Res. Lett.*, *32*, L07801, doi:10.1029/2004GL021189.

1. Introduction

[2] Waves and turbulence are observed at all heights in the atmosphere, and have an important impact on mixing [*Whiteway et al.*, 2003, 2004; *Pavelin et al.*, 2002], cirrus cloud formation [*Gultepe and Starr*, 1995], the stratospheric [*Duck et al.*, 1998, 2001] and mesospheric [*Lindzen*, 1981] circulations, and layered phenomena [e.g., *Duck and Greene*, 2004]. However, the relative contributions of waves and turbulence to the spectrum of atmospheric motions remains uncertain, and the characteristic slope of the horizontal wavenumber spectrum for winds in the lower stratosphere has been variously attributed to both gravity waves and turbulence [*Koshyk and Hamilton*, 2001].

[3] In May and June 2000, new measurements of velocities, temperatures, and composition were obtained at the tropopause above the UK using the Grob G520T 'Egrett' high-altitude research aircraft [*Whiteway et al.*, 2003, 2004]. We present the measurements from a case study on 6 June 2000, which show a variety of motions extending from large-scale waves to small-scale turbulence. Horizontal spectra are determined separately for turbulent and non-turbulent locations. The data were recorded at 1.5 m resolution, and so the spectra extend to the smallest scales yet available at these heights. A wave with 500 m wavelength is identified, and was found in a region of shear instability. The 500 m wave provides a transition between gravity waves and turbulence.

2. Measurements

[4] The flight plan for 6 June 2000 is given by *Whiteway et al.* [2004]. Here we consider measurements along three vertically stacked horizontal flight legs between 53.3°N, 4.9°W and 53°N, 3.4°W, which extends almost 100 km from over the Irish Sea to the Snowdon Mountains of northern Wales. The flight tracks were aligned with the jet stream, which was directed 15° clockwise from East on this day. Measurements of winds were obtained at 55 Hz with a BAT probe and a Rosemount probe (see *Whiteway et al.* [2004] for the experimental details); results from the BAT probe are presented here, and are in good agreement with those from the Rosemount. The aircraft's airspeed averaged 85 m/s, and so the spectrum of gravity waves is not expected to be significantly affected by non-stationarity of the velocity field [*Bacmeister et al.*, 1996].

[5] Figure 1a shows measurements of vertical wind speed fluctuations at the three overlying levels, and Figure 1b shows the terrain height below the flight track. At the lower level (10.3 km), a coherent wave pattern is seen along the entire flight track, and has a mean wavelength of 13.3 km over the ocean; occasional patches of turbulence are also evident. Measurements at the middle level (11.6 km) show strong turbulence everywhere, and the upper level (12.4 km) contains a wavelike structure with only occasional patches of turbulence. An interesting feature in the measurements, shown in the inset of Figure 1a, is the occasional appearance of a small-scale oscillation, hereafter referred to as the "500 m wave".

[6] Measurements were taken during the ascent shown in Figure 1a so that vertical profile of the buoyancy frequency (N) and the Richardson number (Ri) could be determined. In the calculations, the wind and temperature data were first smoothed in the vertical using a 2 km boxcar mean in order to remove the effects of small-scale horizontal variations during the gradual ascent. As is shown in Figure 1c, the lower and middle flight legs were obtained in regions with $Ri \lesssim 0.25$, which are dynamically unstable. Turbulence encountered during the ascent (not shown) was well matched with locations of low Richardson number.

[7] Waves were also observed in the horizontal winds and potential temperatures (not shown). Over the mountains, the oscillations were complicated due to the intrusion of mountain waves. Over the ocean, however, a coherent 13.3 km wave was observed, but with the jet-stream aligned wind and temperature oscillations phase-shifted by 90 degrees in

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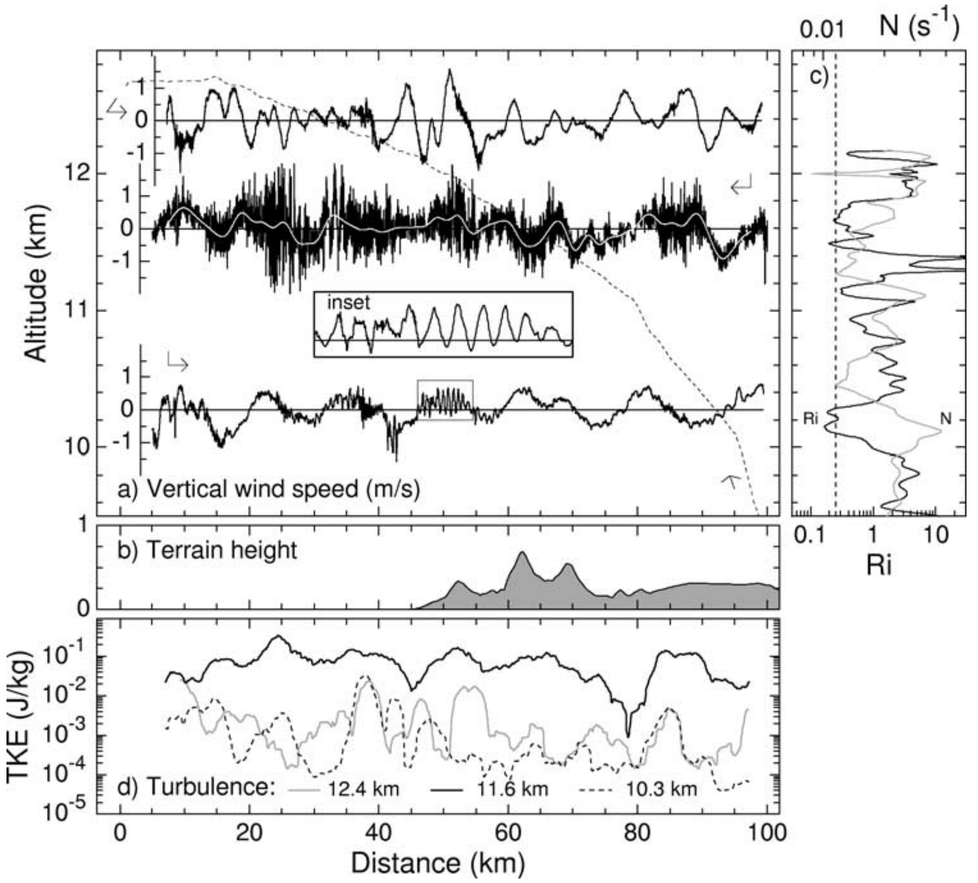

Figure 1. (a) Measurements of vertical wind fluctuations at three vertically stacked levels between 53.3°N, 4.9°W and 53°N, 3.4°W on 6 June 2000 (the inset is for measurements between 46 and 54.5 km distance at the lower level); (b) terrain height along the flight track; (c) the buoyancy frequency (*N*) and the Richardson number (*Ri*); and (d) the 3D turbulent kinetic energy for scales shorter than 100 m, smoothed using a 2 km boxcar window. In Figure 1a, the dashed line shows the initial ascent (30 mins), and the subsequent measurements were obtained in 14, 26, and 13 minute intervals, respectively. The average time between flight tracks was 2.5 minutes, and the average airspeed was 85 m/s. Arrows indicate the direction along each flight leg, and the average horizontal wind speeds (directed from left to right) were 26.2, 19.6, and 43.6 m/s, from top to bottom, respectively.

the upstream direction relative to the vertical winds. For a stationary wave, the frequency $k\bar{u}$ of 0.021 s^{-1} is the same as the buoyancy frequency at that level. The phase relationships and proximity to the buoyancy frequency indicate that the wave is vertically trapped and propagating in the upstream direction relative to the mean wind. The phase relationships for the 500 m wave (not shown) are also suggestive of a vertically trapped disturbance.

[8] Log-averaged horizontal kinetic energy spectra of vertical wind speeds for the three flight tracks are given in Figure 2, and were measured in a frame of reference relative to air by using distances calculated from the aircraft's airspeed. The spectra were determined using the continuous Morlet Wavelet transform, which employs Gaussian localized plane-wave packets as a basis set rather than the infinite plane waves used by the Fourier technique [*Torrence and Compo*, 1998]. Morlet Wavelet spectra are similar to Fourier spectra, but are defined at each point along a data series; adjacent spectra are correlated at wavelength λ if their positions are separated in distance by less than $\sqrt{2}\lambda$.

[9] The spectral curves given in Figure 2 are effectively flat at scales larger than 10 km wavelength, and show a

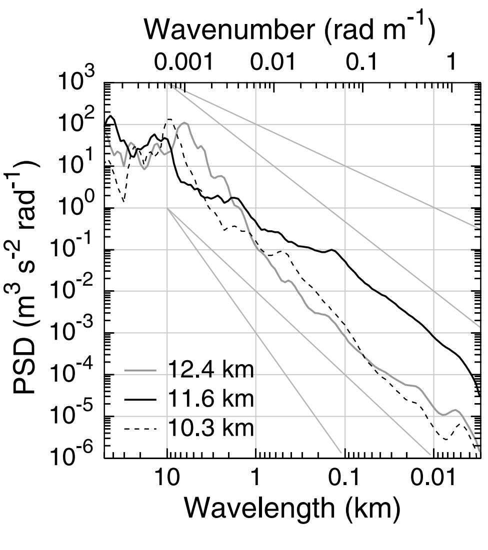

Figure 2. Log-averaged vertical wind kinetic energy spectral densities for each level measured in a frame of reference relative to air. Slanted grey lines show −1, −5/3, −2 and −3 power law dependencies.

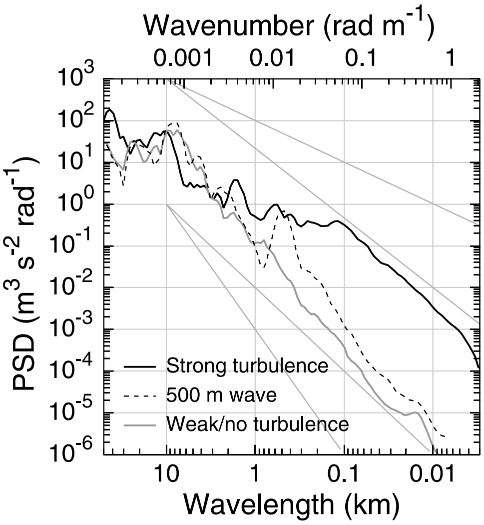

Figure 3. Ensemble-log-averaged vertical wind speed spectra measured in a frame of reference relative to air, obtained in regions of strong turbulence, weak/no turbulence, and strong 500 m waves.

power-law decrease to smaller scales. Below 1 km wavelength, the spectra show significantly different turbulent intensities. The spectrum for the turbulent middle level has a slope of $-5/3$ at small scales. Low-energy bumps at 16 m wavelength and smaller in the relatively non-turbulent upper and lower flight legs are due to the noise floor of the instrument.

[10] Figure 1d gives the turbulent kinetic energy, determined by integrating the wavelet spectra for each component over scales shorter than 100 m. As is shown in Figure 1d, the turbulent kinetic energy is greater in the middle level than in either the upper or lower levels. However, the strength of the turbulence in the middle level varies, and the upper and lower levels show patches of moderate turbulence interspersed in a background containing virtually no turbulence at all.

[11] Figure 3 shows log-averaged vertical wind speed spectra constructed from 2 km wide spectral averages sampled in regions with strong turbulence (7 cases), weak/no turbulence (8 cases), and 500 m waves (7 cases). The turbulent spectra were sampled at locations with peaks in the turbulent kinetic energy for the middle level. The non-turbulent spectra were similarly identified, but from data in the upper and lower levels. The 500 m wave spectra were sampled from the upper and lower levels at locations with clear peaks in the wavelet spectra at 500 m wavelength.

[12] Similar to Figure 2, the spectra given in Figure 3 are essentially flat at scales larger than 10 km. Below 10 km wavelength, the non-turbulent spectrum decays with a -2.75 power-law slope. The turbulent spectrum has a -0.7 power-law slope from 1 km in wavelength to a "knee" at 100 m scale, below which it decays with a -2 power-law slope. The turbulent spectrum at small scales is over 3 orders of magnitude more powerful than the non-turbulent spectrum.

[13] The spectral peak of the 500 m wave has values similar to those found in the turblent spectrum, but at small scales has values similar to the non-turbulent case. The spectrum was measured in a frame of reference relative to air. Relative to ground, the wavelength is 770 m, as may be determined from the inset of Figure 1a.

[14] The ratios of the vertical wind speed kinetic energy spectra to the corresponding total velocity kinetic energy spectra are given in Figure 4. The turbulent spectrum is equipartitioned in three dimensions at 100 m in wavelength; note that this is the same scale as the knee in the turbulent spectrum that was presented in Figure 3. The strong turbulence case has relatively suppressed vertical motions at all other scales. The fall-off of the spectral ratios at scales larger than 10 km is due to the flattening of the vertical wind speed spectra, which is not observed in the horizontal velocity spectra.

[15] For weak turbulence, the predominant oscillations have energy approximately equipartitioned for three dimensional motions. The 500 m wave has energy equally partitioned in two dimensions between the vertical and jet-stream aligned horizontal wind.

3. Discussion

[16] The measurements given in Figure 1a show large-scale oscillations that can be attributed to gravity waves. In particular, the 13.3 km wave in the lower level over the ocean has the phase and frequency characteristics of a trapped wave propagating in the upstream direction relative to the mean wind. A possible source for these waves is mountains in Ireland. A distinct wave with 500 m wavelength was also observed.

[17] The mean spectra presented in Figure 2 are similar to those measured by *Nastrom and Gage* [1985] and *Bacmeister et al.* [1996], although these spectra extend to much shorter scales: 3 m, as opposed to 2 km and 400 m in the earlier studies, respectively. We have observed a flattening of the vertical wind spectrum for scales larger than 10 km, as was seen by *Bacmeister et al.* [1996]. The slopes of the mean spectra for small scales depend very strongly on the amount of turbulence present, with the most turbulent level having a slope of $-5/3$. Note that we have not considered the relative impact of meteorology/season or orography in this case

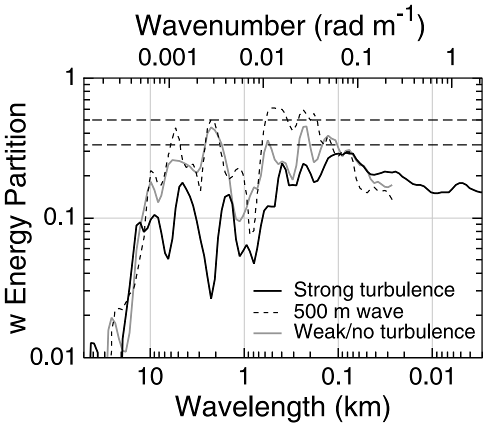

Figure 4. Ratio between mean vertical wind speed spectra and the corresponding total velocity spectra as a function of wavelength. The long-dashed lines indicate kinetic energy equipartition for 3-dimensional motions (i.e., 1/3) and for 2-dimensional motions (i.e., 1/2).

study, which are addressed in the climatological studies of *Nastrom and Gage* [1985] and *Nastrom et al.* [1987].

[18] Average spectra constructed from samples in regions of strong versus weak/no turbulence and given in Figure 3 are similar down to scales of a few kilometers. The spectral slope for the non-turbulent case at small scales is very steep, approaching -3. The lack of turbulence suggests the origin of this spectrum is the atmospheric gravity waves present in the observations. The spectrum for strong turbulence, however, is almost flat between 1 km and 100 m scales, and decays with a -2 power-law slope for the shorter wavelengths. The small-scale turbulent spectrum is over 3 orders of magnitude more energetic than the non-turbulent spectrum.

[19] The appearance of coherent oscillations with 500 m wavelengths is intriguing because this wavelength is between the junction of the turbulent and non-turbulent spectra and the turbulent spectral knee. The 500 m wave provides a transition spectrum between the turbulent and non-turbulent cases: the kinetic energy spectral density at 500 m is similar to that in the turbulent spectrum, and approaches the non-turbulent spectrum at smaller scales. The 500 m wave has a jet-stream aligned horizontal component, but is only weakly evident in the perpendicular direction, which is why kinetic energy in the 500 m wave is equipartitioned for two-dimensional motions in Figure 4.

[20] The shape of the spectrum for strong turbulence is a unique result. At wavelengths shorter than a few kilometers it is nearly flat until the sharp knee at 100 m in scale. Only at the knee is the turbulent kinetic energy equipartitioned in the three dimensions. At both shorter and longer wavelengths the vertical motions are relatively suppressed so that the motions are predominantly in the horizontal plane. This is most likely due to buoyancy, which can quickly flatten initially three-dimensional turbulence [see, e.g., *Lin and Pao*, 1979]. For strong turbulence, the vertical motions at large scales is suppressed relative to the non-turbulent cases. This suggests that turbulence has removed energy predominantly from the vertical motions of the waves.

[21] Phase relationships suggest that the 500 m wave is some form of vertically trapped disturbance. 500 m waves have been observed in association with mountain wave breaking [*Whiteway et al.*, 2003] and Kelvin-Helmholtz instabilities [*Whiteway et al.*, 2004], and were interpreted with respect to counter-rotating shear-aligned rolls. Breakdown of the 500 m mode apparently results in a cascade from 100 m down to smaller scales. The factors that determine the scale separation between the 500 m wave and the knee at 100 m scale in the turbulent spectrum are not known.

[22] The measurements imply that both gravity waves and turbulence play a role at the tropopause. The gravity waves were mostly responsible for motions at scales greater than 1 km, and turbulence, when present, dominated the smaller scales. The measurements suggest that the combination of waves and turbulence of different kinetic energies could produce a variety of power-law slopes in horizontally measured fluctuation spectra. Average slopes of $-5/3$ might only result after considerable averaging, and could potentially remove much of the information on physical processes. For example, the average spectral slope at small scales for the turbulent middle level was $-5/3$, whereas the spectrum for the most turbulent regions had a slope at small scales of -2.

[23] Future work on these measurements will focus on the details of the highly variable two-dimensional wavelet spectra. In particular, co-wavelet spectra will be used to determine the fluxes of both heat and momentum for each scale and as a function of position along the flight track. We will also investigate further the dynamics of the 500 m wave. The measurements will be used to provide a detailed experimental view of turbulence at the tropopause.

[24] **Acknowledgments.** The Aberystwyth Egrett Experiment was funded by the Upper Troposphere/Lower Stratosphere programme of the UK Natural Environment Research Council. The Egrett is owned and operated by Airborne Research Australia at Flinders University in Adelaide, Australia.

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